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DEFORMATION AND FRACTURE MECHANICS

On the Theory of Isothermal Hot Rolling of an Aluminum Alloy Strip

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Abstract—The two-dimensional problem of isothermal (including superplasticity conditions) rolling of an aluminum sheet (at a low angle of bite) is analytically solved, and the influence of high homologous temperature on the pressure on rolls is estimated. The energy–force and kinematic parameters of rolling are found using a dynamic model, which adequately describes the experimental data obtained for commercial aluminum alloys. Calculation results are presented for AMg5 and D18T alloys.

Keywords: rolling, superplasticity, aluminum alloys, strain rate, pressure on rolls

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INTRODUCTION

Technological metal forming processes are usually based on a powerful action on the material to be deformed [1–5]. The unique properties of the metals and alloys, which consist in a sharp decrease in the strain resistance, can be used during metal forming under superplasticity conditions [1, 6]. Here, superplasticity is considered as a specific state of a polycrystalline material, which undergoes plastic deformation at a low stress and has the fine-grained structure having formed at a preliminary stage (structural superplasticity) or during heating and subsequent deformation (dynamic superplasticity) [7]. Both types of superplasticity are characterized by the predominance of grain-boundary sliding over other mass-transfer mechanisms [8, 9]. Obviously, for the dynamic superplasticity effect to occur, an initial (deformed or as-cast) structural state should change into another one to be capable of superplasticity. This change in commercial aluminum alloys takes place in the temperature–rate region of dynamic recrystallization [10–12].

FORMULATION OF THE PROBLEM

We consider the two-dimensional problem of isothermal (including superplasticity conditions [7]) hot rolling of a strip in rolls of radius R rotating at the same angular velocity. Rolling is assumed to occur at a low angle of bite. This means that the investigation [13–15] of the material flow in a wedge converging channel with apex angle α_1 can be used to find the energy–force and kinematic parameters of rolling (Fig. 1).

We introduce cylindrical coordinate system $\rho\alpha z$, place the origin of coordinates at the vertex of a wedge, and direct axis z normal to the figure plane. All geometric sizes are assumed to be divided by the strip width.

The mathematical formulation of the problem in terms of the theory of elastoplastic small-curvature processes [13] includes the following:

differential equilibrium equations

$$\frac{\partial\sigma_\rho}{\partial\rho} + \frac{1}{\rho}\frac{\partial\tau_{\rho\alpha}}{\partial\alpha} + \frac{\sigma_\rho - \sigma_\alpha}{\rho} = 0;$$
$$\frac{\partial\tau_{\rho\alpha}}{\partial\rho} + \frac{1}{\rho}\frac{\partial\sigma_\alpha}{\partial\alpha} + \frac{2\tau_{\rho\alpha}}{\rho} = 0; \quad (1)$$

kinematic relations

$$\dot\varepsilon_\rho = \frac{\partial\upsilon_\rho}{\partial\rho};\ \dot\varepsilon_\alpha = \frac{\upsilon_\rho}{\rho};\ \dot\gamma_{\rho\alpha} = \frac{1}{\rho}\frac{\partial\upsilon_\rho}{\partial\alpha}; \quad (2)$$

incompressibility condition (in velocity)

$$\dot\varepsilon_\rho + \dot\varepsilon_\alpha = \frac{\partial\upsilon_\rho}{\partial\rho} + \frac{\upsilon_\rho}{\rho} = 0; \quad (3)$$

defining relations

$$\sigma_\rho - \sigma_0 = \frac{2\sigma_u}{3\dot\varepsilon_u}\dot\varepsilon_\rho;\quad \sigma_\alpha - \sigma_0 = \frac{2\sigma_u}{3\dot\varepsilon_u}\dot\varepsilon_\alpha \quad (4)$$
$$\tau_{\rho\alpha} = \frac{\sigma_u}{3\dot\varepsilon_u}\dot\gamma_{\rho\alpha};\quad 3\sigma_0 = \sigma_\rho + \sigma_\alpha;$$

equation of state [7] in the form of the dependence of stress intensity σ_u on strain rate intensity $\dot\varepsilon_u$,

$$\sigma_u = 1 - m_0 - \beta + (3m_0 + \beta)\dot\varepsilon_u - 3m_0\dot\varepsilon_u^2 + m_0\dot\varepsilon_u^3. \quad (5)$$

Here, σ_ρ, σ_α, and $\tau_{\rho\alpha}$ are the stress tensor components; $\dot{\varepsilon}_\rho$, $\dot{\varepsilon}_\alpha$, and $\dot{\gamma}_{\rho\alpha}$ are the strain-rate tensor components; υ_ρ is the radial projection of the velocity vector; σ_0 is the average stress; m_0 is the material constant; and β is the temperature-dependent control parameter (it is constant under isothermal conditions and $\beta < 0$ under superplasticity).

In Eqs. (1)–(5), the stress and strain-rate components are attributed to alternative internal state parameters σ^* and $\dot{\varepsilon}^*$ [16–18], and the radial motion velocity vector is attributed to $\dot{\varepsilon}^* b_0$, where b_0 is the strip width. Boundary conditions will be formulated in solving the problem.

RESOLVING FUNCTION

Integrating differential equation (3), we obtain

$$\upsilon_\rho = \frac{k(\alpha)}{\rho}, \tag{6}$$

where $k(\alpha)$ is an unknown function to be determined.

Using solution (6), we can write the strain-rate components as follows:

$$\dot{\varepsilon}_\rho = -\frac{k(\alpha)}{\rho^2};\ \dot{\varepsilon}_\alpha = \frac{k(\alpha)}{\rho^2};\ \dot{\gamma}_{\rho\alpha} = -\frac{k'(\alpha)}{\rho^2}. \tag{7}$$

The strain rate intensity under radial flow is [15]

$$\dot{\varepsilon}_u = \frac{1}{\rho^2} L^{1/2}(\alpha), \tag{8}$$

where

$$L(\alpha) = \frac{1}{3}\left[4k^2(\alpha) + (k'(\alpha))^2\right]. \tag{9}$$

Allowing for equation of state (5) and Eqs. (8) and (9), we write defining relations (4) in the form

$$\sigma_\rho - \sigma_0 = -\frac{2}{3} T(\alpha,\rho)\, k(\alpha);$$
$$\sigma_\alpha - \sigma_0 = \frac{2}{3} T(\alpha,\rho)\, k(\alpha);$$
$$\tau_{\rho\alpha} = \frac{1}{3} T(\alpha,\rho)\, k'(\alpha), \tag{10}$$

where

$$T(\rho,\alpha) = (1 - m_0 - \beta)\, L^{-\frac{1}{2}}(\alpha) + \frac{3m_0 + \beta}{\rho^2} - \frac{3m_0}{\rho^4} L^{\frac{1}{2}}(\alpha) + \frac{m_0}{\rho^6} L(\alpha). \tag{11}$$

An analysis of Eqs. (6)–(11) shows that the stress, velocity, and strain components can be found if an explicit form of function $k(\alpha)$, which is called resolving according to [19], is determined. To search for function $k(\alpha)$, we substitute Eqs. (10) and (11) into equilibrium equation (1). We differentiate the determined derivatives $\partial\sigma/\partial\rho$ and $\partial\sigma/\partial\alpha$ with respect to α and ρ, respectively. We equate the right-hand sides of the obtained mixed derivates to each other. After rather simple but awkward manipulations, which are similar to those in [7], we can show that function $k(\alpha)$ satisfies the differential equation

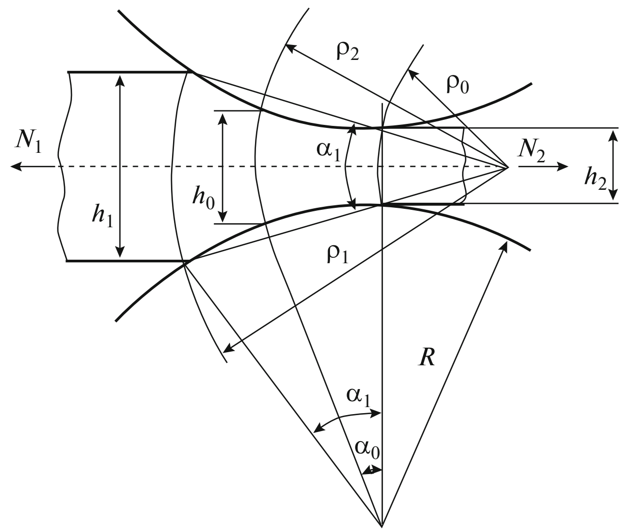

Fig. 1. Geometric scheme for rolling a strip.

$$k'''(\alpha) + 4k'(\alpha) = 0. \tag{12}$$

Allowing for two obvious boundary conditions

$$\tau_{\rho\alpha}\big|_{\alpha=0} = 0;\quad \tau_{\rho\alpha}\big|_{\alpha=\frac{\alpha_1}{2}} = -\chi\tau_{\max}\big|_{\alpha=\frac{\alpha_1}{2}}, \tag{13}$$

we can write the solution to Eq. (12) in the form [20]

$$k(\alpha) = \frac{c}{2}(\psi - \cos 2\alpha), \tag{14}$$

where χ is the experimental coefficient that determines the roll-strip contact conditions [13], c is a constant, and function $\psi(\alpha_1, \chi)$ is

$$\psi(\alpha_1,\chi) = \frac{\sqrt{1-\chi^2}}{\chi}\sin\alpha_1 - \cos\alpha_1. \tag{15}$$

Constant c can be determined simultaneously with the choice of the plastic-deformation zone; according to [13, 19], this zone becomes a wedge bounded by two velocity break surfaces $\rho_1 = \rho_1(\alpha_1)$ and $\rho_2 = \rho_2(\alpha_2)$ at the entrance to the rolls and the exit from them, respectively (see Fig. 1). The procedure of analyzing the deformation zone was comprehensively described in [19] and can be used to find resolving function $k(\alpha)$ and functions $\rho_1(\alpha_1)$ and $\rho_2(\alpha_2)$, which bound the plastic-deformation zone in a radial direction. We have

$$k(\alpha) = \frac{\upsilon_1 h_1}{\overline{\psi}}(\psi - \cos 2\alpha); \tag{16}$$

$$\rho_1(\alpha) = \frac{h_1}{2\overline{\psi}}\frac{2\psi\alpha - \sin 2\alpha}{\sin\alpha};$$
$$\rho_2(\alpha) = (1-\Lambda)\frac{h_1}{2\overline{\psi}}\frac{2\psi\alpha - \sin 2\alpha}{\sin\alpha}. \tag{17}$$

Here, υ_1 is the average strip speed; h_1 and h_2 are the strip thicknesses at the entrance to the rolls and the exit from them, respectively; and $\Lambda = h_2/h_1$ is the reduction of the strip [14, 21] (see Fig. 1)

$$\bar{\psi}(\alpha_1,\chi) = (\psi\alpha_1 - \sin\alpha_1). \tag{18}$$

STATE OF STRESS IN THE PLASTIC-DEFORMATION ZONE

The stress tensor components are determined by solving differential equilibrium equations (1) simultaneously with Eqs. (7), (11), and (14). Explicit expressions for the stress components are given in the form [22] required for the further analysis. We have

$$\begin{aligned}3\sigma_\rho &= (1-m_0-\beta)L^{-1/2}\left(\frac{k'L'}{2L}-k''+4k\right)\\ &\times \ln\frac{\rho}{\rho_2} - 4(1-m_0-\beta)L^{-1/2}k\\ &- \frac{3m_0+\beta}{2}(k''-4k)\left(\frac{1}{\rho_2^2}-\frac{1}{\rho^2}\right) - 4(3m_0+\beta)\frac{k}{\rho_2^2}\\ &+ \frac{3}{4}m_0L^{1/2}\left(\frac{k'L'}{2L}+k''-4k\right)\\ &\times\left(\frac{1}{\rho_2^4}-\frac{1}{\rho^4}\right) + 12m_0L^{1/2}\frac{k}{\rho_2^4} - \frac{m_0}{6}L\\ &\times\left(\frac{k'L'}{L}+k''-4k\right)\left(\frac{1}{\rho_2^6}-\frac{1}{\rho^6}\right) - 4m_0L\frac{k}{\rho_2^6};\end{aligned} \tag{19.1}$$

$$\begin{aligned}3\sigma_\alpha &= (1-m_0-\beta)L^{-1/2}\left(\frac{k'L'}{2L}-k''+4k\right)\\ &\times\ln\frac{\rho}{\rho_2} - \frac{3m_0+\beta}{2}(k''-4k)\left(\frac{1}{\rho_2^2}-\frac{1}{\rho^2}\right)\\ &+ \frac{3}{4}m_0L^{1/2}\left(\frac{k'L'}{2L}+k''+12k\right)\left(\frac{1}{\rho_2^4}-\frac{1}{\rho^4}\right)\\ &- \frac{m_0}{6}L\left(\frac{k'L'}{L}+k''+8k\right)\left(\frac{1}{\rho_2^6}-\frac{1}{\rho^6}\right);\end{aligned} \tag{19.2}$$

$$\begin{aligned}3\tau_{\rho\alpha} &= \Big[(1-m_0-\beta)L^{-1/2}\\ &+ \frac{3m_0+\beta}{\rho^2} - \frac{3m_0}{\rho_4}L^{1/2} + \frac{m_0}{\rho^6}L\Big]k.\end{aligned} \tag{19.3}$$

Here, $L = L(\alpha)$ and $k = k(\alpha)$ are determined by Eqs. (9) and (16), respectively.

Note that Eqs. (19) were derived under the boundary conditions used for the forward creep zone in the plastic-deformation zone, $\rho_0 \le \rho \le \rho_2$, where $\rho_0 = \rho_0(\alpha)$ corresponds to the boundary of the forward and backward creep zones $\rho_0 \le \rho \le \rho_1$.

Naturally, we assume that the longitudinal forces vanish at the entrance to the deformation zone (N_1) and the exit from it (N_2). Therefore, we can write

$$\begin{aligned}N_1 &= 2\int_0^{\alpha_1/2}\sigma_\rho\big|_{\rho=\rho_1}dA = 0;\\ N_2 &= 2\int_0^{\alpha_1/2}\sigma_\rho\big|_{\rho=\rho_2}dA = 0,\end{aligned} \tag{20}$$

where A is the cross-sectional area of the strip to be rolled.

After substituting Eq. (19.1) into the second condition in Eq. (20), it transforms into the cubic equation

$$a_1 + a_1\mu + a_2\mu^2 + a_3\mu^3 = 0, \tag{21}$$

where μ is the parameter that generalizes the velocity, geometric, thermal, and contact factors and is written as

$$\mu = \upsilon_1\left[\frac{\bar{\psi}}{h_1(1-\Lambda)}\right]^2. \tag{22}$$

The coefficients of Eq. (21) are functions of the angle of bite:

$$\begin{aligned}a_0(\alpha_1) &= \frac{\sqrt{3}}{2}(1-m_0-\beta)J_0(\alpha_1);\\ a_1(\alpha_1) &= (3m_0+\beta)J_1(\alpha_1);\\ a_2(\alpha_1) &= -2\sqrt{3}m_0J_2(\alpha_1);\\ a_3(\alpha_1) &= \frac{4}{3}m_0J_3(\alpha_1),\end{aligned} \tag{23}$$

where

$$\begin{aligned}J_0(\alpha_1) &= \int_0^{\alpha_1/2}H^{-1/2}(\alpha)H_1(\alpha)H_2(\alpha)d\alpha;\\ J_1(\alpha_1) &= \int_0^{\alpha_1/2}H_1(\alpha)H_2^{-1}(\alpha)d\alpha;\\ J_2(\alpha_1) &= \int_0^{\alpha_1/2}H^{1/2}(\alpha)H_1(\alpha)H_2^{-3}(\alpha)d\alpha;\\ J_3(\alpha_1) &= \int_0^{\alpha_1/2}H(\alpha)H_1(\alpha)H_2^{-5}(\alpha)d\alpha,\end{aligned} \tag{24}$$

Here, we have

$$\begin{aligned}H(\alpha) &= 1+\psi^2-2\psi\cos 2\alpha;\\ H_1(\alpha) &= \psi - \cos 2\alpha;\\ H_2(\alpha) &= \frac{2\psi\alpha - \sin 2\alpha}{2\psi\sin\alpha}.\end{aligned} \tag{25}$$

Figure 2 shows the results of numerical solution of Eq. (21) as the graphical dependences of parameter μ on the angle of bite α_1 at the temperatures inside ($\beta < 0$) and outside ($\beta > 0$) the superplasticity range of AMg5 and D18T aluminum alloys [7]. The calculations were carried out at $\chi = 0.3$ and $m_0 = 0.3333$ (AMg5 alloy) and $m_0 = 0.3965$ (D18T alloy).

We now analyze the backward creep zone. The first condition in Eq. (20) leads to the integral

$$\int_0^{\alpha_1/2}\left[(1-m_0-\beta)L^{-1/2}\left(\frac{k'L'}{2L}-k''+4k\right)\ln\rho_1\right.$$
$$+\frac{3m_0+\beta}{2\rho_1^2}(k''-4k)-\frac{3}{4}m_0\frac{L^{1/2}}{\rho_1^4}\left(\frac{k'L'}{2L}+k''-4k\right)\quad(26)$$
$$\left.+m_0\frac{L}{6\rho_1^6}\left(\frac{k'L'}{L}-k''+4k\right)+D\_(\alpha)\right]\rho_1 d\alpha=0,$$

where $D\_(\alpha)$ is an arbitrary function belonging to the boundary between the forward and backward creep zones.

Allowing for arbitrary notations (25), we present integral (26) in the form

$$F(\alpha_1)=-\int_0^{\alpha_1/2}D\_(\alpha)H_2(\alpha)d\alpha,\qquad(27)$$

where

$$F(\alpha_1)=2\sqrt{3}(1-m_0-\beta)Q_0(\alpha_1)+2(3m_0+\beta)$$
$$\times\frac{\upsilon_1\bar{\Psi}}{h_1}Q_1(\alpha_1)-2\sqrt{3}m_0\frac{\upsilon_1^2\bar{\Psi}^2}{h_1^2}Q_2(\alpha_1)\qquad(28)$$
$$+\frac{8}{9}m_0\frac{\upsilon_1^3\bar{\Psi}^3}{h_1^3}Q_3(\alpha_1),$$

Here, we have

$$Q_0(\alpha_1)$$
$$=\int_0^{\alpha_1/2}\left[\frac{\psi\sin^2 2\alpha}{H^{3/2}(\alpha)}-\frac{H_3(\alpha)}{H^{1/2}(\alpha)}\right]H_2(\alpha)\ln\left[\frac{h_1}{\bar{\Psi}}H_2(\alpha)\right]d\alpha;$$

$$Q_1(\alpha_1)=\int_0^{\alpha_1/2}\frac{H_3(\alpha)}{H_2(\alpha)}d\alpha;$$

$$Q_2(\alpha_1)=\int_0^{\alpha_1/2}\frac{H^{1/2}(\alpha)}{H_2^3(\alpha)}\left[\frac{\psi\sin^2 2\alpha}{H(\alpha)}+H_3(\alpha)\right]d\alpha;\quad(29)$$

$$Q_3(\alpha)=\int_0^{\alpha_1/2}\frac{H(\alpha)}{H_2^5(\alpha)}\left[\frac{2\psi\sin^2 2\alpha}{H(\alpha)}+H_3(\alpha)\right]d\alpha;$$

$$\frac{\upsilon_1\bar{\Psi}}{h_1}=\mu\frac{h_1}{\bar{\Psi}}\left[\frac{2R}{h_1}(1-\cos\alpha_1)\right]^2.\qquad(30)$$

Recall that υ_1 is the average velocity of a strip of thickness h_1 at the entrance to the deformation zone, α_1 is the angle of bite, and R is the roll radius.

The calculations performed in [15] for AMg5 and D18T alloys revealed that the left-hand side of equality (27) could be approximated by a quadratic function; therefore, we can write

$$b\alpha_1^2+p\alpha_1=-\int_0^{\alpha_1/2}D\_(\alpha)H(\alpha)d\alpha.\qquad(31)$$

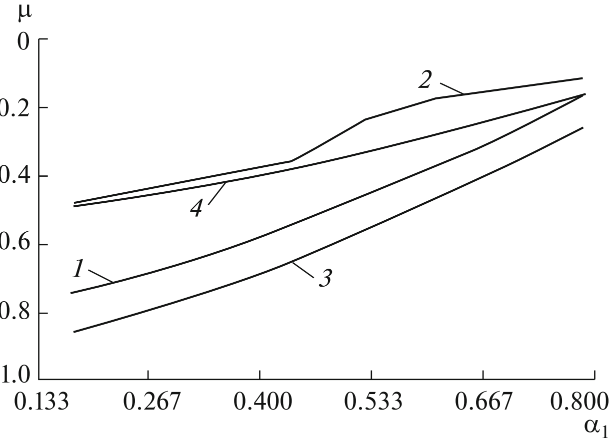

Fig. 2. Parameter μ vs. angle of bite α_1 for (*1*, *2*) AMg5 and (*3*, *4*) D18T alloys at the following rolling parameters: (*1*) $\beta=-0.134$ (753 K), (*2*) $\beta=0.231$ (693 K), (*3*) $\beta=-0.127$ (793 K), and (*4*) $\beta=0.123$ (713 K).

Differentiating both sides of equality (31) with respect to $\alpha_1/2$, we obtain

$$D\_(\alpha_1/2)=-(2b\alpha_1+p)H_2^{-1}(\alpha_1/2),\qquad(32)$$

where

$$H_2(\alpha_1/2)=\frac{\psi\alpha_1-\sin\alpha_1}{2\sin(\alpha_1/2)}.\qquad(33)$$

Note that coefficients b and p depend on material constants, thickness h_1, and contact coefficients χ.

Allowing for Eq. (32), we write formulas for determining the normal stresses at the material–tool contact (at $\alpha=\alpha_1/2$) in the backward creep zone as follows:

$$3\sigma_\rho|_{\alpha=\alpha_1/2}=(1-m_0-\beta)L^{-1/2}\left(\frac{k'L'}{2L}-k''+4k\right)$$
$$\times\ln\rho+\frac{3m_0+\beta}{2\rho^2}(k''-4k)-\frac{3}{4}m_0\frac{L^{1/2}}{\rho^4}$$
$$\times\left(\frac{k'L'}{2L}+k''-4k\right)+\frac{m_0L}{6\rho^6}\left(\frac{k'L'}{L}-k''+4k\right)\qquad(34.1)$$
$$-(2b\alpha_1+p)\frac{\bar{\Psi}\rho_1}{h_1};$$

$$3\sigma_\alpha=(1-m_0-\beta)L^{-1/2}$$
$$\times\left[\left(\frac{k'L'}{2L}-k''+4k\right)\ln\rho+4k\right]$$
$$+\frac{3m_0+\beta}{2\rho^2}(k''-4k)-\frac{3}{4}m_0\frac{L^{1/2}}{\rho^4}\qquad(34.2)$$
$$\times\left(\frac{k'L'}{2L}+k''-20k\right)$$
$$+\frac{m_0L}{6\rho^6}\left(\frac{k'L'}{L}-k''+28k\right)-(2b\alpha_1+p)\frac{\bar{\Psi}\rho_1}{h_1}.$$

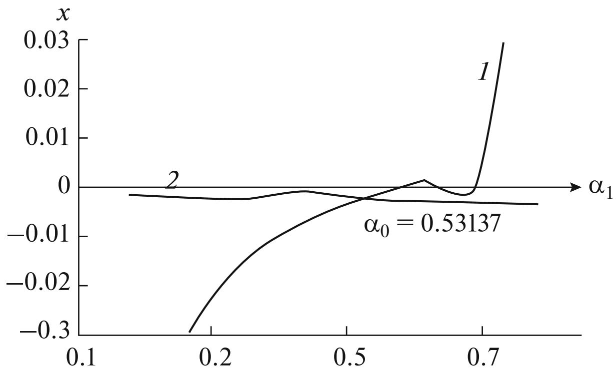

Fig. 3. Parameter x vs. angle of bite α_1 for an AMg5 alloy: (*1*) calculation by Eq. (37) and (*2*) calculation by Eq. (40).

BOUNDARY BETWEEN THE FORWARD AND BACKWARD CREEP ZONES

The condition for determining the boundary between the forward and backward creep zones is the equality

$$\sigma_\rho\big|_{\rho=\rho_0+0} = \sigma_\rho\big|_{\rho=\rho_0-0}. \tag{35}$$

Using Eqs. (19.1) and (34.1), at the strip–roll contact ($\alpha = \alpha_1/2$) we have

$$S_0(\alpha_1) + S_1(\alpha_1)x + S_2(\alpha_1)x^2 + S_3(\alpha_1)x^3 = 0. \tag{36}$$

Here, we have

$$x = \frac{\upsilon_1 h_1}{\bar{\psi}}; \tag{37}$$

$$\begin{gathered} S_0(\alpha_1) = 2\sqrt{3}(1 - m_0 - \beta)L^{-1/2} \\ \times\left[\left(\frac{\psi\sin^2\alpha_1}{H^{3/2}(\alpha_1)} - \frac{H_3(\alpha_1)}{H^{1/2}(\alpha_1)}\right)\ln\rho_2 + H_1(\alpha_1)\right] \\ -(2b\alpha_1 + p)H_2(\alpha_1); \\ S_1(\alpha_1) = \frac{2(3m_0+\beta)}{\rho_2^2}\psi; \\ S_2(\alpha_1) \\ = -2\sqrt{3}\frac{m_0}{\rho_2^4}\left[\frac{\psi\sin^2\alpha_1}{H^{1/2}(\alpha_1)} + (3\psi - 2\cos\alpha_1)H^{1/2}(\alpha_1)\right]; \\ S_3(\alpha_1) = \frac{8}{9}\frac{m_0}{\rho_2^6} \\ \times\left[2\psi\sin^2\alpha_1 + (5\psi - 4\cos\alpha_1)H(\alpha_1)\right]; \end{gathered} \tag{38}$$

and

$$\begin{gathered} H(\alpha_1) = 1 + \psi^2 - 2\psi\cos\alpha_1; \\ H_1(\alpha_1) = \psi - \cos\alpha_1; \\ H_2(\alpha_1) = \frac{\psi\alpha_1 - \sin\alpha_1}{2\sin(\alpha_1/2)}; \\ H_3(\alpha_1) = 2\cos\alpha_1 - \psi; \\ \rho_2(\alpha_1) = \frac{h_1}{\bar{\psi}}(1-\Lambda)\frac{\psi\alpha_1 - \sin\alpha_1}{2\sin(\alpha_1/2)}; \end{gathered} \tag{39}$$

and functions $\psi = \psi(\alpha_1, \chi)$ and $\bar{\psi} = \bar{\psi}(\alpha_1, \chi)$ are determined by Eqs. (15) and (18), respectively.

On the other hand, based on Eq. (30), for $x = \upsilon_1 h_1/\bar{\psi}$ we can write

$$\frac{\upsilon_1 h_1}{\bar{\psi}} = \mu\left(\frac{h_1}{\bar{\psi}}\right)^3\left[\frac{2R}{h_1}(1-\cos\alpha_1)\right]^2. \tag{40}$$

Figure 3 shows parameter x calculated by Eq. (37) as a function of angle of bite α_1 (curve *1*) at a strip thickness $h_1 = 0.07$ m at the entrance to the deformation zone and constants $m_0 = 0.3333$, $\chi = 0.3$, $R = 0.11$ m, and $\beta = -0.134$ (AMg5 alloy) [7, 15]. Curve *2* was plotted using Eq. (40). The point of intersection corresponds to angle $\alpha_0 = 0.53137$ rad, which corresponds to the boundary between the forward and backward creep zones at the alloy–tool contact (at $\alpha = \alpha_1/2$).

At the found value of angle α_0, neutral layer thickness h_0 can be determined using the law of changing the strip thickness in the deformation zone [19],

$$h_0 = (1-\Lambda)h_1 + 2R(1-\cos\alpha_0). \tag{41}$$

Using Eq. (17), we write the radius of the neutral layer at $\alpha = \alpha_1/2$ as

$$\rho_0\left(\frac{\alpha_1}{2}\right) = \frac{h_0}{\bar{\psi}}\frac{\psi\alpha_1 - \sin\alpha_1}{2\sin\frac{\alpha_1}{2}}. \tag{42}$$

Calculations were performed for AMg5 and D18T alloys inside and outside the superplasticity temperature range at three initial strip thicknesses $h_1 = 0.03$, 0.07, and 0.1 m and $\chi = 0.3$, 0.4, and 0.45. The choice of thickness h_1 is caused by relation to a low-grade two-high 220 rolling mill [23]. The calculation results demonstrate that the entire deformation zone can be considered as a forward creep zone at low values of h_1 and coefficient χ. An increase in parameters h_1 and χ causes the appearance of backward and forward creep zones, and a stick zone appears at high values of h_1 and χ. In this case, the flat cross-section hypothesis, which is the basis for the rolling model, is known to be violated [19].

FORCE PARAMETERS OF ROLLING

We identify pressure on rolls $q = q(\rho)$ with the circumferential normal stress at the contact surface [24],

$$\sigma_\alpha\big|_{\alpha=\alpha_1/2} = -q. \tag{43}$$

Quantity $\sigma_\alpha\big|_{\alpha=\alpha_1/2}$ is determined by Eq. (34.2) for the backward creep zone and by Eq. (19.2) for the forward creep zone ($\rho_0 \le \rho \le \rho_2$). Thus, we obtain

$$q = -\frac{1}{3}\begin{cases} 2\sqrt{3}(1-m_0-\beta)E(\alpha_1/2)\ln\dfrac{\rho(\alpha_1/2)}{\rho_2(\alpha_1/2)} - 2(3m_0+\beta)\left[\dfrac{1}{\rho_2^2(\alpha_1/2)} - \dfrac{1}{\rho^2(\alpha_1/2)}\right]\psi\dfrac{\upsilon_1 h_1}{\overline{\psi}} \\ + 2\sqrt{3}\left[\dfrac{1}{\rho_2^4(\alpha_1/2)} - \dfrac{1}{\rho^4(\alpha_1/2)}\right]H^{1/2}(\alpha_1/2)\left[\dfrac{\psi\sin^2\alpha_1}{H(\alpha_1/2)} + 3\psi - 2\cos\alpha_1\right]\left(\dfrac{\upsilon_1 h_1}{\overline{\psi}}\right)^2 \qquad (44.1) \\ - \dfrac{8}{9}m_0\left[\dfrac{1}{\rho_2^6(\alpha_1/2)} - \dfrac{1}{\rho^6(\alpha_1/2)}\right]H(\alpha_1/2)\left[\dfrac{2\psi\sin^2\alpha_1}{H(\alpha_1/2)} + 2\psi - \cos\alpha_1\right]\left(\dfrac{\upsilon_1 h_1}{\overline{\psi}}\right)^3 \text{ at } \rho_2 \le \rho \le \rho_0; \\ 2\sqrt{3}(1-m_0-\beta)\left[E(\alpha_1/2)\ln\rho(\alpha_1/2) + \dfrac{\psi-\cos\alpha_1}{H^{1/2}(\alpha_1/2)}\right] + \dfrac{2(3m_0+\beta)}{\rho^2(\alpha_1/2)}\psi\dfrac{\upsilon_1 h_1}{\overline{\psi}} \\ - 2\sqrt{3}\dfrac{H^{1/2}(\alpha_1/2)}{\rho^4(\alpha_1/2)}\left[\dfrac{\psi\sin^2\alpha_1}{H(\alpha_1/2)} + 6\cos\alpha_1 - 5\psi\right]\left(\dfrac{\upsilon_1 h_1}{\overline{\psi}}\right)^2 + \dfrac{8}{9}m_0\dfrac{H(\alpha_1/2)}{\rho^6(\alpha_1/2)} \qquad (44.2) \\ \times\left[\dfrac{2\psi\sin^2\alpha_1}{H(\alpha_1/2)} + 7\psi - 8\cos\alpha_1\right]\left(\dfrac{\upsilon_1 h_1}{\overline{\psi}}\right)^3 - (2b\alpha_1 + p)H_2(\alpha_1/2) \text{ at } \rho_0 \le \rho \le \rho_1. \end{cases}$$

Here, $H(\alpha_1/2)$, $H_1(\alpha_1/2)$, $H_2(\alpha_1/2)$, $H_3(\alpha_1/2)$, and $\rho_2(\alpha_1/2)$ are determined by the formulas

$$E(\alpha_1/2) = \frac{\psi\sin^2\alpha_1}{H^{3/2}(\alpha_1/2)} - \frac{H_3(\alpha_1/2)}{H^{1/2}(\alpha_1/2)}. \qquad (45)$$

which are analogous to Eq. (39).

Note that Eq. (44.1) is used to calculate the pressure on rolls in the case where the forward creep zone occupies the entire deformation zone.

Figure 4 shows the dependences of the pressure on rolls q on radius $\rho = \rho(\alpha_1/2)$ that were plotted using Eqs. (44.1) and (44.2) for AMg5 and D18T alloys at the constants used earlier. We can conclude that a temperature from the superplasticity range (743–773 K for the AMg5 alloy and 783–823 K for the D18T alloy) favors a sharp decrease in the pressure on rolls (by a factor of 5–7), all other thing being equal. This conclusion is supported by the $q(\rho)$ curves. The calculations were carried out at an angle of bite $\alpha_1 = \pi/12$, an initial strip thickness $h_1 = 0.07$ m, and parameter $\chi = 0.3$. Isotherms *1–4* of both alloys correspond to the temperatures at which the $\mu(\alpha_1)$ were plotted (see Fig. 2).

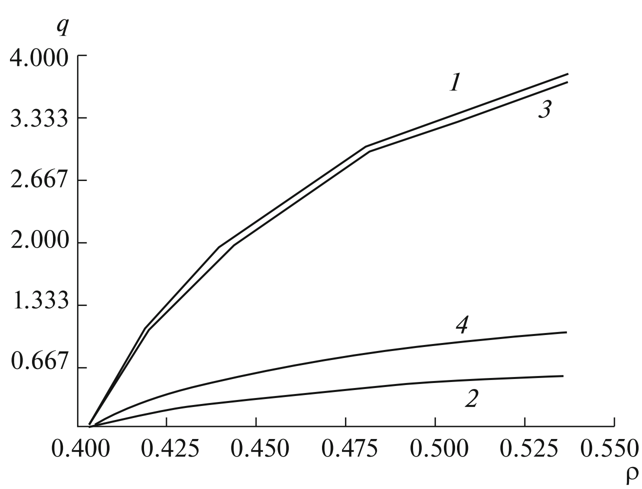

Fig. 4. Pressure on rolls q vs. radius ρ for (*1*, *2*) AMg5 and (*3*, *4*) D18T alloys at the following rolling parameters: (*1*) $\beta = 0.231$ (693 K), (*2*) $\beta = -0.134$ (753 K), (*3*) $\beta = 0.123$ (713 K), and (*4*) $\beta = -0.127$ (793 K).

The technological parameters of rolling are seen to be sensitive to the initial grain size. For example, the grain size is 4–7 μm in both alloys under temperature–rate superplasticity conditions [23], and the grain size does not exceed 45 μm in the as-deformed state of the AMg5 alloy and reaches 130 μm in the D18T alloy. Therefore, the grain refinement in the D18T alloy is more substantial than in the AMg5 alloy. Hence, rolling of the D18T alloy in the superplasticity range occurs under more rigid conditions (at higher pressures) as compared to the AMg5 alloy.

CONCLUSIONS

We analytically solved the two-dimensional boundary problem of rolling steel sheets under isothermal conditions, including the superplasticity temperature ranges. Explicit expressions were derived to determine the velocity, the strain, the stress field, and the pressure on rolls. The pressure on rolls was shown to depend on the thermomechanical parameters of the material to be deformed, the rate of feed of a strip into rolls, and the conditions in the strip–roll contact region.

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